Dark Matter as Dimensional Condensate

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Abstract

Fractional statistics (FS) is a generalization of the spin-statistics theorem and mixes bosons and fermions in a non-trivial way. Mixing is controlled by a continuous parameter $0 \leq q \leq 1$ and the ordinary statistics is recovered in the limit $q = 1$. We have argued some time ago that the onset of FS occurs in a spacetime endowed with minimal fractality, whose ground state is the *Cantor Dust*, an early Universe phase created by topological condensation of continuous dimensions. Recent studies on q- bosons reinforce the hypothesis that Dark Matter is the relic of Cantor Dust left over from the early stages of cosmological evolution. The take-away point of this brief note is the growing support for the minimal fractality of spacetime and its ramifications in foundational physics.

Key words: Dark Matter, minimal fractal manifold, Cantor Dust, *q*-bosons, fractional statistics, dimensional condensate.

The *spin-statistics theorem* is a fundamental principle of quantum physics and reflects the contrasting behavior of bosons and fermions in three-dimensional space. There are various extensions of the theorem enabling bosons and fermions to overlap and they are referred to as fractional statistics, anyon statistics and quantum groups [4]. These extensions have found a broad range of applications from deformed algebras of q-bosons and q-fermions to non-commutative field theory, cosmic strings, and Black Holes, to fractional quantum Hall effect and anyonic states of matter [1, 4-6]. The algebra of q - particles is specified by the following set of commutation relationships for the ladder operators $a, a^\dagger$ and the number operator N [2]

$$a a^\dagger - q^{\pm 1} a^\dagger a = q^{\pm N} \tag{1}$$

$$\left[N, a^\dagger \right] = a^\dagger \tag{2a}$$

$$\left[N, a \right] = -a \tag{2b}$$

By (1) and (2), the Fock eigenstates $|n\rangle$ are built as in

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{[n]!}} |0\rangle, \quad a|0\rangle = 0 \tag{3}$$

where the q-basic number and factorial are defined as, respectively,

$$[x] = \frac{q^{x} - q^{-x}}{q - q^{-1}} \tag{4}$$

$$[n]! = [n][n-1]...[1] \tag{5}$$

Ordinary numbers x correspond to the limit $q \to 1$, that is,

$$\lim_{q \to 1}[x] = x \tag{6}$$

The action of the operators on the state $|n\rangle$ is given by

$$a^{\dagger}|n\rangle = [n+1]^{1/2}|n+1\rangle \tag{7a}$$

$$a|n\rangle = [n]^{1/2}|n-1\rangle \tag{7b}$$

$$N|n\rangle = n|n\rangle \tag{7c}$$

The Hamiltonian operator of a q-deformed harmonic oscillator is shown to take the form

$$H = \frac{\hbar\omega}{2}(a\,a^{\dagger} + a^{\dagger}a) \tag{8a}$$

leading to the following spectrum of eigenvalues on the basis $|n\rangle$

$$E(n)=\frac{\hbar\omega}{2}([n]+[n+1]) \tag{8b}$$

A close relationship exists between *fractional differential operators* and *q*-deformed algebras [2]. To fix ideas, consider the power function

$$f(x,\alpha)=x^{n\alpha} \tag{9}$$

in which α is the index of fractional differentiation. Setting

$$\alpha=1-\varepsilon,\ |n,\varepsilon\rangle=f(n,\varepsilon)=x^{n(1-\varepsilon)} \tag{10}$$

yields the following expression of the Caputo fractional derivative of (10)

$$D_x^{1-\varepsilon}|n,\varepsilon\rangle=\frac{\Gamma[1+n(1-\varepsilon)]}{\Gamma[n(1-\varepsilon)+\varepsilon]}|n,\varepsilon\rangle; \quad n>0 \tag{11}$$

which, in turn, leads to

$$[n]_{1-\varepsilon}|n,\varepsilon\rangle=D_x^{1-\varepsilon}|n,\varepsilon\rangle \tag{12}$$

and

$$\lim_{\varepsilon \to 0} \left[n \right]_{1-\varepsilon} = n \qquad (13)$$

It follows from (6) and (13) that the direct dentification

$$\boxed{q = 1 - \varepsilon,\ 0 \le q \le 1} \qquad (14)$$

connects *fractional statistics* to field theory built on fractional differential operators (called *fractional field theory* [7]). Moreover, *minimal fractal manifold* (MFM) describes a scale-dependent spacetime equipped with low-level fractality, where the continuous deviation from integer dimensionality assumes the form

$$\varepsilon(\mu) = \frac{m^2(\mu)}{\Lambda_{UV}^2} << 1 \qquad (15)$$

Here, μ, m, Λ_{UV} denote the running scale, mass parameter and ultraviolet cutoff, respectively. At the far ultraviolet end of the energy scale $m = O(\Lambda_{UV})$ both q and spacetime dimensionality drop to zero, a condition akin to the Planckian regime of *spacetime singularities*.

Remarkably, recent modeling [3] shows that *q*-bosons offer an intriguing picture of Dark Matter (DM), as *q*-bosons freeze in a condensed phase, regardless of temperature. We have suggested some time ago that the onset of fractional statistics naturally develops on the minimal fractal manifold (MFM), whose ground state is the *Cantor Dust*, an early Universe phase generated by *topological condensation of continuous dimensions*. These findings reinforce the conjecture that DM represents a relic of Cantor Dust left over from the early stages of cosmological evolution [7-9]. It is also instructive to note that the concept of Cantor Dust may enable an unforeseen unification of DM and Dark Energy [10], as well as a platform for reconciling the particle physics and gravitational interpretations of DM [11].

References

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